

Pirie-Kieren-Based Integrative Language Literacy and Numeracy Model in Junior High School Geometry

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Abstract: This article formulates an integrative model of language literacy and numeracy for analysing junior high school students' conceptual understanding of geometry through the Pirie-Kieren framework. The study uses a descriptive-exploratory qualitative design, with 60 Grade VII students at SMPN 21 Pontianak in the initial mapping stage and six purposively selected students for in-depth analysis. Data are obtained through conceptual understanding tests, literacy-based tasks, classroom observations, semi-structured interviews, and learning artefacts. Open, axial, and selective coding are combined with triangulation and mapping onto the eight Pirie-Kieren layers. The resulting model connects text comprehension, verbal-visual-symbolic representations, numerical reasoning, folding back, and conceptual-linguistic-representational-numerical obstacles in learning the surface area of polyhedra. The main finding is that geometric understanding is a dynamic coordination of language, representation, and numeracy rather than formula mastery alone. Theoretically, the model extends the Pirie-Kieren framework by positioning language literacy and numeracy as constitutive mechanisms within each layer of understanding and by providing operational indicators for tracing students' movement and folding back across layers.

Keywords: conceptual understanding, geometry learning, language literacy, numeracy, Pirie-Kieren theory

INTRODUCTION

Conceptual understanding is a fundamental basis for learning mathematics because it enables students to grasp the meaning of concepts, recognise connections among ideas, and apply knowledge in different situations. In geometry learning, conceptual understanding becomes even more important because students must connect shapes, measurements, positions, relationships among elements, visual representations, and spatial reasoning. Students' difficulties in geometry are evident not only in procedural errors but also in their inability to explain why a formula is used, identify the elements of a geometric figure, and connect two-dimensional figures with three-dimensional solids.

Recent scholarship confirms that geometry learning requires the coordination of visualisation, spatial reasoning, and domain-specific geometric knowledge. Harris (2023) and Hawes and Ansari (2020) show that spatial reasoning supports how learners represent, organise, and

transform mathematical objects, while Atit et al. (2020) report that spatial skills and mathematics motivation jointly predict middle school mathematics achievement. Fujita et al. (2020) further demonstrate that students solving two-dimensional representations of three-dimensional shapes must coordinate spatial visualisation with property-based reasoning, and Adams et al. (2023) show that spatial training can transfer to measurement and geometry performance. These findings establish the visual-spatial demands of geometry, but they do not by themselves explain how language, numerical reasoning, and the layered movement of understanding operate together.

Difficulties in geometry are also closely related to the coordination of representations. Duval (2017) explains that mathematical thinking depends on transformations among registers such as natural language, figures, symbols, and procedures. In tasks on the surface area of polyhedra, students must interpret verbal information, recognise solids or nets, identify relevant measurements, and connect them to symbolic formulas. Failure may therefore arise from an inability to move flexibly among representations rather than from computation alone.

Language is equally central to mathematical learning. Erath et al. (2021) show that instruction which deliberately develops mathematical language supports participation, explanation, and concept formation. Ufer and Bochnik (2020) distinguish the contributions of general and subject-specific language skills, while Schüller-Meyer et al. (2019) demonstrate that multilingual language resources can support conceptual understanding. Prediger (2019) and Zahner et al. (2021) further show that language-responsive instruction requires teachers to identify content-specific language demands and design opportunities for mathematical discourse. Recent synthesis also positions language, multimodal communication, and mathematical meaning making as intertwined research domains (Planas & Pimm, 2024). These findings indicate that language literacy and numeracy should be treated as interacting resources in geometry learning.

The Pirie-Kieren framework views mathematical understanding as dynamic, layered, recursive, and not always linear. Gulkilik et al. (2020) demonstrate how the framework can trace growth across a multi-representational learning environment. The eight layers range from primitive knowing, image making, and image having to property noticing, formalising, observing, structuring, and inventising. Yao and Manouchehri (2022) further show that folding back to earlier forms of knowing can support reflection and the construction of mathematical generalisations in a dynamic geometry environment.

An integrated diagnostic model is urgent because contemporary mathematics education expects students to reason, communicate, coordinate representations, and interpret mathematics in context (OECD,

2023). However, classroom diagnosis often still reduces geometry difficulty to incorrect formula use. Language-responsive research likewise shows that robust understanding depends on mathematical richness, cognitive demand, discourse, equitable participation, and explicit connections among registers (Prediger & Neugebauer, 2021). When these dimensions are assessed separately, teachers may prescribe repeated calculations even when the underlying obstacle is linguistic, representational, numerical, or conceptual. An integrated model is therefore needed to locate both the level of understanding and the source of difficulty before an intervention is selected.

The research gap lies in the tendency of previous studies to examine spatial reasoning, mathematical representation, language, numeracy, or the growth of understanding as separate constructs. Recent reviews still identify fragmented taxonomies of spatial ability (Schenck & Nathan, 2024), while language-focused research commonly concentrates on discourse, multilingual learning, or teacher expertise without integrating numerical and layered conceptual development (Planas & Pimm, 2024; Prediger, 2019). Few analytical frameworks explain how these dimensions interact at each Pirie-Kieren layer when junior high school students work with the surface area of polyhedra. The novelty of this article is a single integrative model that maps language literacy and numeracy onto the eight layers and uses folding back to interpret conceptual, linguistic, representational, and numerical obstacles. Accordingly, this article aims to formulate an operational analytical model that helps researchers and teachers identify students' positions of understanding and select interventions that address the actual source of difficulty.

METHOD

This study uses a qualitative descriptive-exploratory design because it seeks a detailed account of students' thinking, mathematical language, coordination of representations, and obstacles during the construction of geometric understanding (Creswell & Poth, 2018). Operationally, the design comprises six connected stages: problem orientation, initial mapping, participant selection, in-depth exploration, integrative analysis, and validation with model formulation.

Stage 1, problem orientation, delineates the surface area of polyhedra as the content focus, aligns the conceptual-understanding and literacy indicators with the research questions, and prepares the test, observation, interview, and artefact-analysis guides. Stage 2, initial mapping, involves 60 Grade VII students at SMPN 21 Pontianak completing a conceptual-understanding test and being observed during instruction. The results are described to identify variations in verbal, visual, symbolic, and numerical understanding.

Stage 3, participant selection, uses the mapping results to purposively select six students who represent varied trajectories of understanding. Stage 4, in-depth exploration, gathers complementary evidence through literacy-based tasks, classroom observations, semi-structured interviews, and students' learning artefacts. The tasks examine how students read and restate information, shift between text, figures, nets, symbols, and calculations, and explain the reasoning behind their choices.

Stage 5, integrative analysis, begins with descriptive analysis of the 60 students' test responses and continues with open coding of the six selected students' verbal, written, and visual data. Axial coding connects codes into conceptual, linguistic, representational, and numerical categories, while selective coding identifies the core relationships among categories (Saldaña, 2021). The categories are then mapped onto the eight Pirie-Kieren layers to identify positions of understanding, movements across layers, and folding-back patterns.

Stage 6 combines validation and model formulation. Trustworthiness is strengthened through technique and source triangulation, member checking of interview interpretations, analytical discussions among the research team, and an audit trail linking claims to tests, tasks, observations, interviews, and artefacts. Convergent patterns are formulated as model components and analytical indicators; divergent cases are retained to refine the boundaries of each indicator and prevent premature generalisation.

Table 1. Focus of Research Questions, Data Sources, and Analysis

Research Focus	Data Sources	Analytical Direction
Development of conceptual understanding in geometry	Concept tests, observations, interviews, and learning artefacts	Movement of understanding across the eight Pirie-Kieren layers and the emergence of folding back
Contribution of language literacy to the construction of understanding	Literacy-based tasks and interviews	Ability to read problems, understand terms, formulate explanations, and provide justification
Relationship between language literacy and mathematical representation	Written work, figures, and interview transcripts	Transitions among verbal, visual, symbolic, and numerical representations
Conceptual and linguistic obstacles	Observations, test responses, interviews, and learning artefacts	Conceptual misconceptions, inaccurate terminology, representational errors, and inappropriate use of formulas

DISCUSSION

Integrative Model of Language Literacy and Numeracy

The main result of this article is an integrative model of language literacy and numeracy based on Pirie-Kieren theory for analysing junior high

school students' conceptual understanding of geometry. The model emphasises that understanding the surface area of polyhedra is developed not merely through mastery of formulas but through the coordination of text comprehension, visual representation, symbol use, numerical reasoning, and the ability to communicate mathematical justification. Thus, the study offers an analytical framework that interprets students' errors as multidimensional phenomena rather than solely as procedural mistakes.

Within the Pirie-Kieren framework, language literacy and numeracy can be observed at each layer of understanding. At primitive knowing, students use everyday language and prior experiences of shapes. At image making and image having, students construct drawings, models, or nets. At property noticing, students identify the properties of solids and relationships among measurements. At formalising, they connect verbal explanations with symbols and formulas. At observing, structuring, and inventising, students reflect on procedures, organise conceptual structures, and develop generalisations. This result confirms that language literacy and numeracy are not supplementary elements but integral to the growth of geometric understanding.

Integrative Model of Language Literacy and Numeracy in the Pirie-Kieren Framework

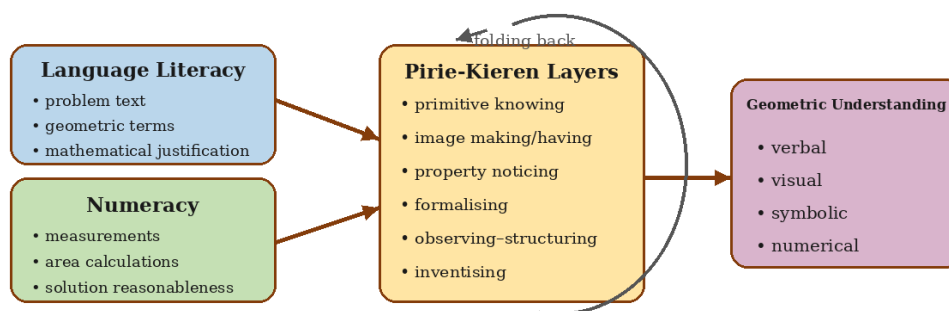


Figure 1. Integrative model of language literacy and numeracy in the Pirie-Kieren framework

Integrated Indicators of the Layers of Understanding

Table 2. presents the findings as a set of analytical indicators. Each Pirie-Kieren layer is mapped onto three dimensions: geometric understanding, language literacy, and numeracy. This mapping provides a practical tool for teachers to trace students' positions of understanding, for example, whether they are still recognising shapes, beginning to construct images, identifying geometric properties, or already using symbols and formulas meaningfully. Using these indicators, teachers can determine more appropriate interventions than simply assigning repetitive calculation exercises.

Table 2. Integrated Analysis Indicators Based on Pirie-Kieren Theory

Layer	Geometric Understanding	Language Literacy	Numeracy
Primitive knowing	Recognising shapes and previously learned concepts of area	Using informal language to explain objects	Recognising numbers, measurements, and basic information
Image making	Constructing drawings, models, or nets	Reading and restating information in the problem	Determining the measurements needed for the solution
Image having	Using mental images of three-dimensional solids	Describing geometric objects orally or in writing	Matching measurements to the appropriate elements of a solid
Property noticing	Identifying faces, edges, planes, and relationships among parts	Naming and explaining the properties of a solid accurately	Connecting properties of the solid with quantitative relationships
Formalising	Using symbols and formulas with clear meaning	Translating verbal information into symbolic form	Applying procedures for calculating surface area
Observing	Reviewing solution procedures and results	Presenting reasoning coherently and logically	Checking the reasonableness of results and units
Structuring	Organising concepts into coherent understanding	Constructing a complete mathematical explanation	Comparing strategies and relationships among results
Inventising	Generating new strategies or generalisations	Communicating original mathematical ideas	Extending numerical reasoning to other contexts

Mapping Conceptual, Linguistic, Representational, and Numerical Obstacles

The next result is a map of conceptual, linguistic, representational, and numerical obstacles in geometry learning. Conceptual obstacles occur when students cannot distinguish surface area from volume or fail to identify all faces of a solid. Linguistic obstacles arise when students misunderstand terms such as face, base, height, plane, or key information in a problem. Representational obstacles occur when students cannot connect text, figures, nets, symbols, and calculations, while numerical obstacles appear in selecting measurements, calculating face areas, or checking units.

This conceptual finding is supported by Duval (2017), who emphasises that mathematical understanding requires coordination across semiotic registers. It is also consistent with Erath et al. (2021), Ufer and Bochnik (2020), and Zahner et al. (2021), who position language as a resource for participation, text comprehension, mathematical explanation, and concept development. The proposed model extends these perspectives by incorporating numeracy and folding back as analytical mechanisms, so

students' errors can be interpreted as interactions among concepts, language, representations, and numerical reasoning.

Table 3. Mapping of Possible Obstacles in Geometry Learning

Type of Obstacle	Indications in Students' Responses	Instructional Intervention
Conceptual	Students do not distinguish surface area from volume; the faces of the solid are not fully identified.	Use concrete objects, nets, and discussion of conceptual meaning before introducing formulas.
Linguistic	Students misunderstand terms such as face, base, height, plane, or key information in the problem.	Guide students to mark keywords, restate the problem, and explain geometric terminology.
Representational	Students cannot connect text, figures, nets, symbols, and calculations.	Practise transitions among verbal, visual, and symbolic representations in gradual stages.
Numerical	Students select incorrect measurements, miscalculate face areas, or do not check units.	Encourage checks of relevant measurements, calculation strategies, estimation, and reasonableness of results.

Pedagogical Implications of the Model

The pedagogical implication is the need for geometry instruction oriented towards meaning making rather than formula rehearsal. Because understanding develops recursively, difficulty should prompt carefully designed folding back to drawings, concrete models, informal explanations, or simpler numerical cases before students return to symbols and generalisations. Yao and Manouchehri (2022) similarly show that folding back can stimulate reflection and support the construction of mathematical generalisations in a dynamic geometry environment.

The findings are consistent with Gulkilik et al. (2020), who use the Pirie-Kieren framework to trace growth in a multi-representational environment, and with the OECD (2023) emphasis on reasoning, formulating, employing, and interpreting mathematics in context. They also complement Nabilah et al. (2024), who report that students with stronger mathematical connections demonstrate more complete mathematical-literacy processes. Recent evidence further indicates that spatial performance improves when targeted reasoning is embedded in geometry instruction (Adams et al., 2023) and that language-responsive teaching benefits from explicit connections across representations and classroom discourse (Prediger & Neugebauer, 2021; Zahner et al., 2021). The principal distinction of this article is that language literacy and numeracy are explicitly mapped within each layer of junior high school geometric understanding.

Theoretically, the model makes three contributions. First, it extends the Pirie-Kieren framework by treating language literacy and numeracy as

mechanisms through which each layer becomes observable, not as external supporting skills. Second, it connects semiotic conversion with folding back, explaining how difficulty in one representation may trigger productive reconstruction at an earlier layer. Third, it operationalises these relationships as indicators that distinguish conceptual, linguistic, representational, and numerical obstacles while preserving their interaction.

Pedagogically, the model supports deep and responsive geometry learning. Teachers can use problem texts, concrete objects, nets, diagrams, symbols, calculations, estimation, unit checks, and oral or written justification as coordinated learning experiences. The indicators help teachers decide whether a student needs support in interpreting a term, constructing an image, connecting representations, selecting measurements, or reflecting on a procedure, thereby enabling more targeted intervention than repetitive calculation exercises.

CONCLUSION

Three principal findings emerge from this article. First, geometric understanding is constructed through the coordination of text comprehension, verbal-visual-symbolic representations, and numerical reasoning across the Pirie-Kieren layers. Second, folding back functions as a productive mechanism for rebuilding meaning when students encounter conceptual, linguistic, representational, or numerical obstacles. Third, the integrated indicators make it possible to locate both a student's layer of understanding and the dominant source of difficulty, so geometry errors are not reduced to computational failure alone.

Theoretically, these findings extend the Pirie-Kieren framework by specifying language literacy and numeracy as constitutive and observable mechanisms within each layer and by linking semiotic conversion to folding back. Pedagogically, the model guides teachers to combine reading, concrete and visual models, symbolisation, meaningful calculation, reasonableness checks, and mathematical justification in targeted interventions. The model is intended for analytical transfer rather than statistical generalisation; future research should test its indicators with broader samples, different geometry topics, and classroom interventions to examine reliability, usability, and effects on conceptual understanding.

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